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Methodology for teaching the buck converter: step by step description of the design

Metodología para la enseñanza del convertidor reductor: descripción paso a paso del diseño

MUÑOZ-GALEANO, Nicolás 1; LOPEZ-LEZAMA, Jesús M. 2 & VILLADA-DUQUE, Fernando 3

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ABSTRACT:
This paper presents a methodology for teaching the buck converter with losses for the power electronics and related courses that can be used for academic purposes and a powerful tool of design. The methodology includes the design details and the stationary state analysis which permits to establish the gains of the system. There are also included efficiency and conduction modes analyses. There were deduced mathematical models that are also useful as a base for scientific papers, validated through simulation results.
Keywords: Buck converter, stationary state, gain of voltage and current, efficiency

RESUMEN:
Este artículo presenta una metodología para la enseñanza del convertidor reductor con pérdidas para el curso de electrónica de potencia y afines que puede usarse con propósitos académicos y como una potente herramienta de diseño. La metodología incluye el detalle del diseño que incluye el análisis del estado estacionario del convertidor que permite establecer las ganancias del sistema. También es incluido un análisis de la eficiencia y de los modos de conducción. Se deducen modelos que también sirven como base de artículos científicos que son validados mediante resultados de simulación.
Palabras clave: Convertidor reductor, estado estacionario, ganancias de voltaje y corriente, eficiencia

1. Introduction

Most of the papers related with the design of buck converters make their contribution in the use of the model, not presenting enough attention in the deduction of the model. Details of the deduction of the model permit the understanding of the operating principle and also facilitate the design of the control structure.

Steady-state analysis of buck converter is useful since it is possible to observe the behavior of the converter in operation, so decisions can be taken from the design stage (Liu*et al*., 2007). Steady-state analysis receives a special attention of designers, being presented as a powerful tool in Liang and Tseng (2005). The deduction of the equations is not an easy task, so most designers prefer to skip the steady-state stage and proceed to the control stage, losing the opportunity to understand the behavior of the converter and making better decisions from the beginning of the design (Arango *et al*., 2013).

In general terms, losses should be taken into account to correctly predict the behavior in the entire operation range of the converter (Beldjajev, and Roasto, 2012; Davoudi *et al.*, 2013). If losses are not included, the steady analysis is straightforward but does not correctly represent the behavior. Simplified models are not able to predict voltage gain and efficiency since they do not consider non-linearities (Galigekere and Kazimierczuk, 2012). Inductor and capacitors losses are usually modeled as series resistances (Geyer *et al.*, 2008; Arango *et al*., 2013; Vlad *et al.*, 2014). Some designing approaches take into account the losses resistances in inductors and capacitors (Geyer *et al*., 2008; Beldjajev and Roasto, 2012; Galigekere and Kazimierczuk, 2012; Davoudi *et al*., 2013). When these series resistances are included in the modeling, the equations obtained from the average model contain from the beginning the non-linearities.

Buck converters are widely used in industry. A development of a buck converter for photovoltaic applications is presented in Hasaneen and Elbaset (2008) and in Masri *et al*. (2012). Deekshitha and Shenoy (2017) presented the design and simulation of synchronous buck converter for LED application. Besides, Bi and Xia (2010) presented a modeling and simulation of dual-mode buck converter. Also, Alargt *et al*., (2017) designed an adaptive delta modulation controller for interleaved buck DC-DC converter. All these works have a steady-state analyses, nonetheless they do not include inductor and capacitor losses.

On the other hand, technical literature is mainly focus on control techniques for buck converters (Chen and Chen, 2008; Restrepo *et al*., 2013; Van der Broeck *et al*., 2015). Others are focused on the design for a particular application (Hasaneen, B. M. and Elbaset, A. A., 2008; Bi, Z. and Xia, W., 2010; Masri, S. et al., 2012; Alargt, *et al*., 2017; Deekshitha C. and Shenoy, K. L., 2017). Technical literature is focus on using the buck converter model for control or development purposes; nevertheless, there is not reported a procedure or methodology to obtain its model or operating principle. The objective of the work is to propose a methodology for teaching step by step the design of a buck converter that can be used for academic purposes and is presented as a powerful tool of design.

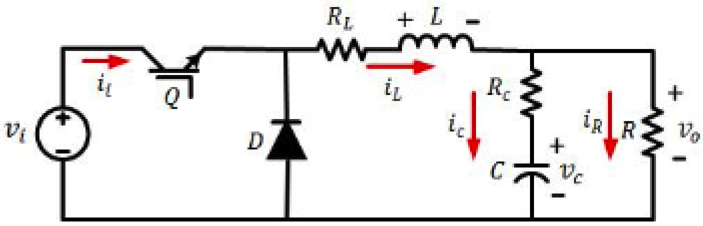
2. Proposed methodology for teaching the buck converter

An understandable methodology for teaching the buck converter must at least include: 1) explanation of switching states and principle of operation, 2) deduction of the differential equations that govern the behavior of the converter, 3) application of average model to obtain the gains of the system to obtain the steady state of the converter, 4) deduction of the transformer model to obtain the efficiency, 5) deduction of the condition to operate in Continuous Condition Mode (CCM). Also, inductor and capacitor losses must be included to obtain a more realistic approximation of the converter. The proposed methodology is composed of five steps:

2.1. Step 1: Establish switching states and explain the principle of operation

Buck converter function is to maintain a regulated output voltage lower than its input voltage; for this reason it is also known as the reducer converter. Figure 1 illustrates the equivalent circuit of the Buck converter. is the input voltage or source, while is the output voltage. The inductor () of the converter stores energy and then transfers it to the capacitor () and the load (). Power transistor () and diode () operate as switches. Diode must be of high speed to prevent that the switching frequency of the power transistor affects its activation time. represents the losses of the inductor, while represents the losses of the capacitor. and are included in the equivalent circuit to obtain a more realistic modeling of the system.

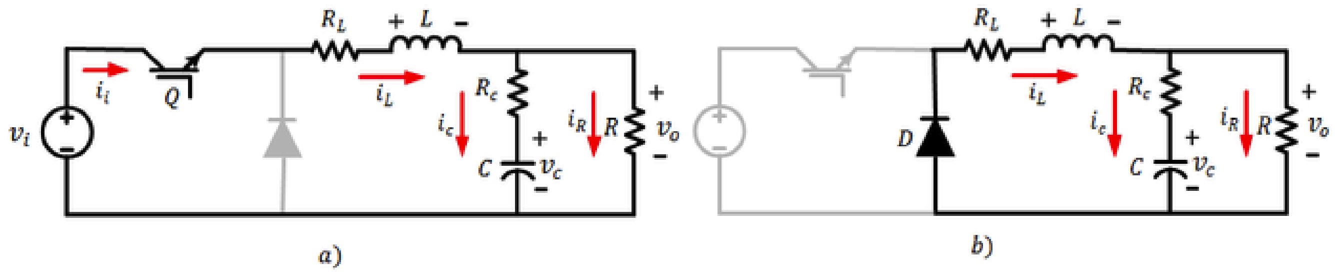
Figure 1
Equivalent circuit of a buck converter



The converter has two switching states according to the position of Q: a) closed and b) open (Figure 2). When is closed (Figure 2a), the diode is reverse polarized, that is, it is open. In this state, the inductor is in series with and the current flows from the input source to the capacitor, charging the inductor while the capacitor supplies power to the load. In this switching state, the current in the inductor increases. When the switch is open (Figure 2b), the source does not feed the converter, the inductor changes its polarity and prevents a sudden change in the inductor current, also the diode is directly polarized and the energy stored in the inductor is transferred to the load and the capacitor through . Passive law of signs should be established to obtain polarities (reference voltages) and senses of currents (reference currents).

Figure 2

States of buck converter: a) closed, b) open



2.2. Step 2: Deduce the differential equations that govern the behavior of the converter.

For writing the differential equations that govern the behavior of the converter, firstly the differential equations for each switching state must be written. Kirchhoff Laws of voltage and current are used taking into account voltage and current references that were established in step 1. It is very important to use the references for writing because, if they are not used, the model would not be coherent. Equations (1) and (2) are the differential equations that govern the behavior when the switch is closed, while equations (3) and (4) are the differential equations operating when the switch is open.

$$L\frac{di_L}{dt} = v_e - v_S - i_L R_L \tag{1}$$

$$C\frac{dv_c}{dt} = i_L - \frac{v_s}{R} \tag{2}$$

$$L\frac{di_L}{dt} = -v_S - i_L R_L \tag{3}$$

$$C\frac{dv_c}{dt} = i_L - \frac{v_s}{R} \tag{4}$$

Switching function u is defined to conveniently write equations (1) to (4). When $u = 1$, the switch is closed; whereas when $u = 0$,the switch is open. Equations (1) and (3) are rewritten as follows:

$$L\frac{di_L}{dt} = (v_e - v_S - i_L R_L)u + (-v_S - i_L R_L)(1-u) = v_e u - v_S - i_L R_L \tag{5}$$

While equations (2) and (4) are rewritten as follows:

$$C\frac{dv_c}{dt} = (i_L - \frac{v_s}{R})u + (i_L - \frac{v_s}{R})(1-u) = i_L - \frac{v_s}{R} \tag{6}$$

For convenience, equations (5) and (6) are rewritten in terms of the system states (i_L and v_c), so the output voltage is expressed using Kirchhoff voltage law as follows:

$$v_s = v_c + i_c R_c = v_c + C\frac{dv_c}{dt}R_c \tag{7}$$

Replacing equation (7) into equation (6) and obtaining $C\frac{dv_c}{dt}$ we have:

$$C\frac{dv_c}{dt} = \left(\frac{R}{R+R_c}\right)i_L - \frac{v_c}{R+R_c} \tag{8}$$

Replacing equations (7) and (8) into equation (5) we have:

$$L\frac{di_L}{dt} = v_e u - \left(\frac{RR_c}{R+R_c} + R_L\right)i_L - \left(\frac{R}{R+R_c}\right)v_c \tag{9}$$

Equations (8) and (9) correspond to the system model in terms of the state variables.

2.3. Step 3: Apply average model to obtain the gains of the system

The average model allows to obtain the steady state of the system, that is, the values of voltage and current in which the system stabilizes. To apply the average model, equations (8) and (9) must be equated to zero. Average model does not take into account fast dynamics (ripples), being slow dynamics the average of the signals. The average of the signals is defined by: the average current of the inductor (I_L), the average voltage of the capacitor (V_c) and the average value of the function u (D) that is called duty cycle.

$$0 = \left(\frac{R}{R+R_c}\right) I_L - \frac{V_c}{R+R_c} \tag{10}$$

$$0 = V_e D - \left(\frac{RR_c}{R+R_c} + R_L\right) I_L - \left(\frac{R}{R+R_c}\right) V_c \tag{11}$$

The following equations are obtained by replacing $I_L = I_i/D$ (The input current I_i is only equal to the current I_L when the switch is closed) and dividing the expression (11) by D.

$$0 = \left(\frac{R}{R+R_c}\right) I_e/D - \frac{V_c}{R+R_c} \tag{12}$$

$$0 = V_e - \left(\left(\frac{RR_c}{R+R_c} + R_L\right)/D^2\right) I_e - \left(\frac{R}{R+R_c}\right) V_c/D \tag{13}$$

The following are the gains of the system and are obtained from (12) and (13). In addition, $\alpha_c = R_c/R$ and $\alpha_L = R_L/R$ are defined to facilitate the writing and the explanation of the equations.

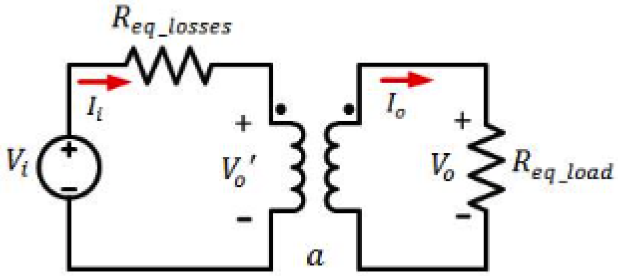
$$H(D) = \frac{V_c}{I_e} = \frac{R}{D} \tag{14}$$

$$G(D) = \frac{V_c}{V_e} = \left(\frac{1}{1+\alpha_L}\right) D \tag{15}$$

2.4. Step 4: Get the transformer model to obtain the efficiency

The transformer model of the converter (Figure 3) is a graphical representation of equations (16) and (17) which are derived from equations (12) and (13). The transformation model is a steady-state circuit that allows the modeling of the converter which not includes the switch. There are defined the following terms: transformation ratio ($a = \left(\frac{R}{R+R_c}\right)/D$), load equivalent resistance ($R_{eq\_load} = R + R_c$) and losses equivalent resistance ($R_{eq\_losses} = \left(\frac{RR_c}{R+R_c} + R_L\right)/D^2$). Relationships that allow transforming voltage and current are also defined ($I_o = aI_i$, $V_o' = aV_o$). For the transformer model, equation (16) represents the sum of currents in the output of the transformer and is drawn in the right side of the transformer; while equation (17) represents the sum of voltages at the input of the transformer and is drawn in the left side of the transformer. It is possible to affirm that the R_{eq_load} includes the resistance R_c which also behaves as the load of the converter; while R_{eq_losses} includes the resistances R, R_c, R_L and D, so R_{eq_losses} concentrates all losses of the device in a single component and depends on the point of operation because it depends on D. The top points in the winding of the transformer, in this case, indicate that the converter does not change the polarity between its input and its output.

Figure 3
Transformer model of buck converter



$$0 = I_s - \frac{V_c}{R_{eq\_carga}} \tag{16}$$

$$0 = V_e - R_{eq\_per} I_e - V_c' \tag{17}$$

The efficiency (η) can be obtained using its classical definition as the relationship between the output power (P_o) and the input power (P_i) ($\eta = P_o/P_i$):

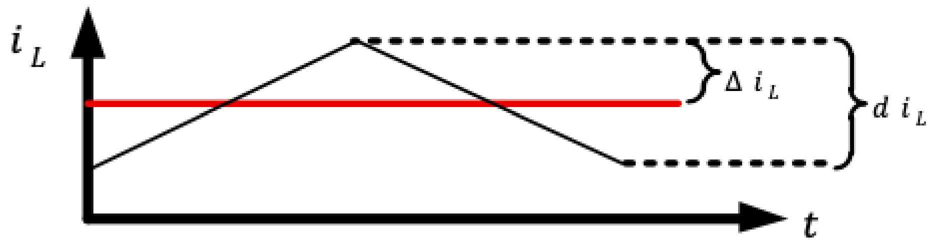
$$\eta = \frac{P_s}{P_e} = \frac{V_s}{V_e} \cdot \frac{I_s}{I_e} = G(D) \cdot a = \frac{1}{(1+\alpha_c)(1+\alpha_L)} \tag{18}$$

Equation (18) was properly represented to facilitate interpretation, using the voltage gain $G(D)$ and the transformation ratio a which is also the relationship between the output and the input currents. For this converter, the efficiency does not depend on duty, depending only of the losses resistance and the load.

2.5. Step 5: Deduce the condition to operate in Continuous Condition Mode (CCM)

CCM facilitates the control of the device and reduces the possibility of electromagnetic interference that affects its performance or the performance of other equipments. To achieve this, it is necessary to maintain the instantaneous current of the inductor with a value greater than zero. Therefore, the CCM occurs when the average current of the inductor is greater than the current ripple ($|I_L| > |\Delta i_L|$), see Figure 4.

Figure 4
Graphical representation of CCM



To obtain the current ripple, it is used equation (1) that corresponds to the equation that governs the behavior of the inductor when the switch is closed. Equation (1) should be conveniently transformed by changing the differentials by discrete values as follows: 1) The current differential must be twice the current delta to be in accordance with the definition of current ripple that is given in this document and shown in Figure 4 ($di_L = 2\Delta i_L$). 2) The time differential is replaced by the time delta when the switch is closed, being expressed in terms of D and the switching period (T_s). Also the instantaneous values of equation (1) are changed to average values. The expression derived from equation (1) and that allows to obtain the CCM mode is:

$$I_L > (V_e - V_S - I_L R_L)\frac{DT_s}{2L} \tag{19}$$

By replacing equation (16) in equation (19) and manipulating it, it is obtained:

$$\left(\frac{2L}{R_{eq\_load}DT_s} + \alpha_L + 1\right) > \frac{V_e}{V_s} = \frac{1}{G(D)} = \frac{(1+\alpha_L)}{D} \tag{20}$$

In the inequality, the constant terms are left on the right side and D, that varies during the operation, is left on the left side as follows:

$$D > 1 - \frac{2Lf_{sw}\eta}{R} \tag{21}$$

Where f_{sw} is the switching frequency ($f_{sw} = \frac{1}{T_s}$). Left side of equation (21) is named in this paper as $K(D)$, while right side of equation (21) is named K_c. To guarantee CCM the inequality of equation (21) must be satisfied ($K(D) > K_c$).

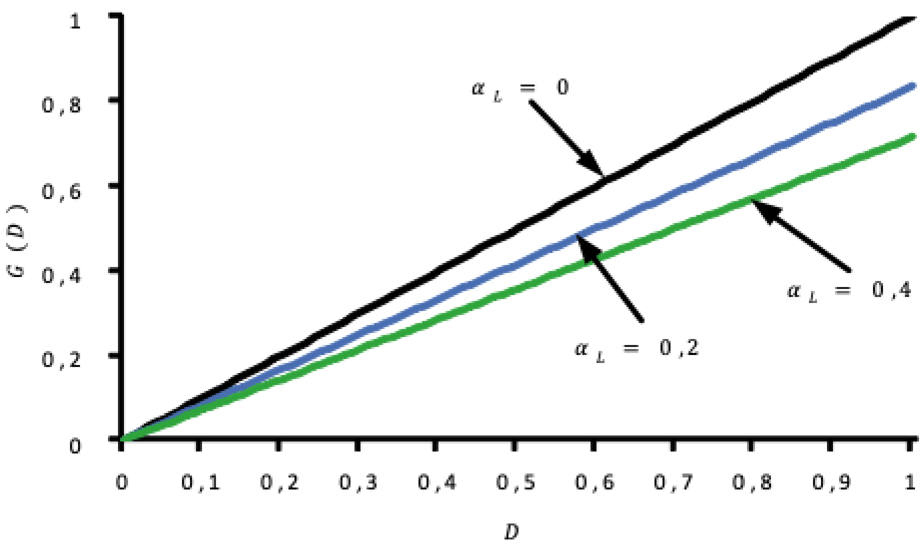
3. Simulation results

It is considered a buck converter with the following specifications: 1) L=5mH, $C = 1200\mu F$, $R = 500\Omega$, $f_{sw} = 20KHz$. For the following analysis, it is considered different values of α_L (0; 0,2; 0,4) and α_c (0; 0,2; 0,4). Specifications of voltages, currents and powers are not necessary due to the results are expressed in term of its gains.

3.1. Gain Voltage

Gain of voltage $G(D)$ is represented in order to observe the behavior of the output voltage when the duty cycle is changed. It is considered the following values of α_L (0; 0,2; 0,4). α_c is not considered in this analysis due to $G(D)$ does not depend on α_c being logic due to the capacitor is not between the input and the output of the converter. Figure 5 shows $G(D)$ vs D.

Figure 5
Gain of voltage for buck converters

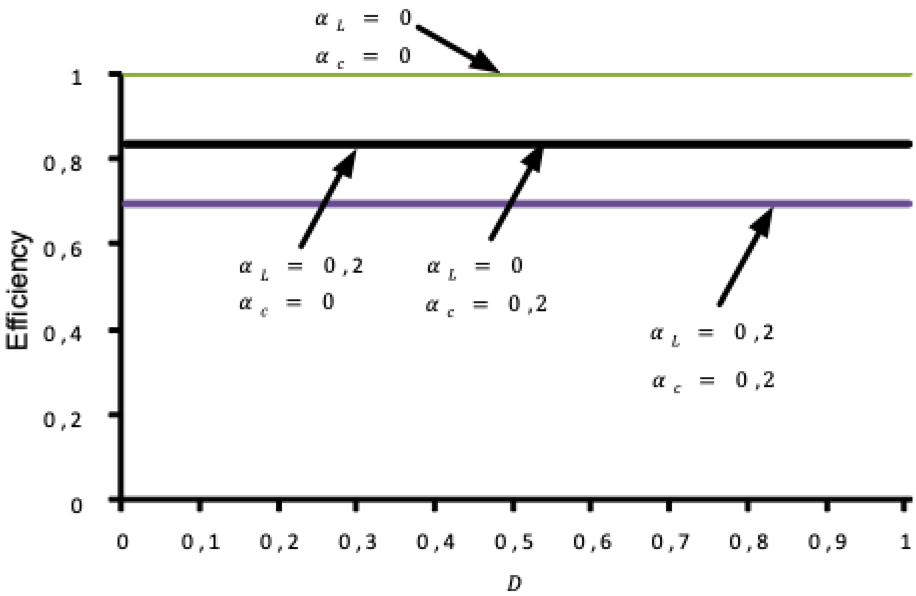


$G(D)$ increases when D is increased. There are represented three curves: the first curve when α_L is zero correspond to a buck converter without losses; it is possible to observe that the gain of voltage can be set between 0 and 1; so that, it is confirmed that the buck converter is a voltage reducer. As α_L increases the voltage range in the output is reduced, as it can be seen with the curves with α_L equal to 0,2 and 0,4. A buck converter with high inductance resistance has a low voltage gain range.

3.2. Efficiency

Efficiency is represented in Figure 6. Efficiency analysis was made considering combinations of the following values of α_L (0; 0,2; 0,4) and α_c (0; 0,2; 0,4). It is observed that in general terms the efficiency is constant not depending on D. The maximum value of efficiency is reached when α_L and α_c are zero, corresponding to an ideal buck converter. The effect of α_L and α_c on the efficiency is equal as can be seen in the curves when ($\alpha_L = 0{,}2$; $\alpha_c = 0$) and ($\alpha_L = 0$; $\alpha_c = 0{,}2$) that are superposed in Figure 6. The combined effect of α_L and α_c can be seen in the curve when ($\alpha_L = 0{,}2$; $\alpha_c = 0{,}2$) with resulting low efficiency.

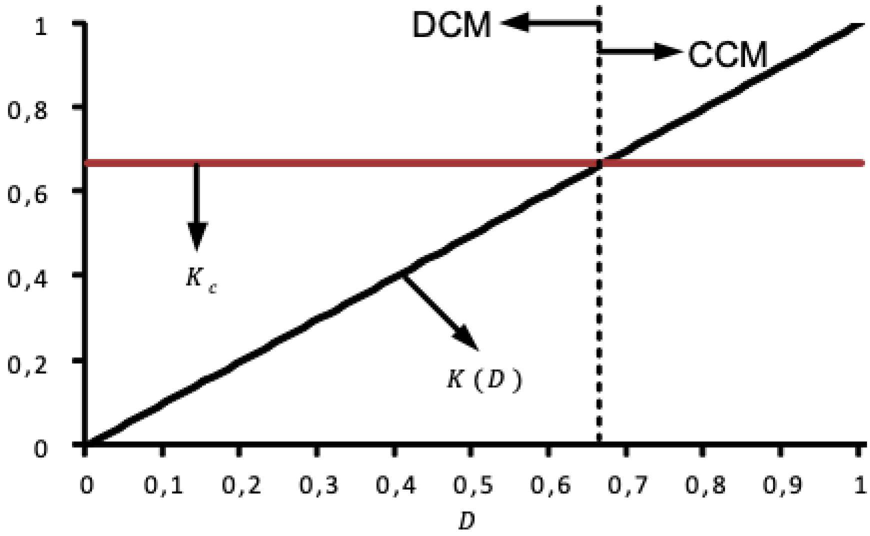
Figure 6
Efficiency for different values of a_L and a_C



3.3. CCMcondition analysis

Figure 7 shows the condition that permits the evaluation of CCM in the buck converter. Losses were set in ($\alpha_L = 0{,}2$; $\alpha_c = 0{,}2$), thus the corresponding efficiency is 0,83. CCM condition is satisfied when $K(D) > K_c$, thus vertical dotted line separates Discontinuous Conduction Mode (DCM) and CCM modes. Left side correspond to DCM, while right side correspond to CCM. For the converter selected few values of D can be used if CCM is desired. For improving the range of CCM switching frequency, inductance, load resistance and losses should be considered from the design stage.

Fig 7
CCM condition for the buck converter



4. Conclusions

This paper presented a methodology for teaching the design of a buck converter with five steps that permits the understanding of its operating principle and main particularities. All equations were carefully organized to ensure that the reader can follow a step by step deduction of equations which facilitates their understanding.

Step one established the switching states; so polarities of voltages and direction of currents were obtained, being possible to establish voltage and current references as well as the buck converter principle of operation.

Step two, following Kirchhoff laws for each switching state, presented the deduction of the differential equations that govern the behavior of the converter. Equations were conveniently written using a switching function.

Step three used the average model to obtain the gains of the system. This allows to establish the steady state of the system, that is, the values of voltage and current in which the system stabilizes.

Step four presented the transformer model to obtain the efficiency of the converter. Average model were conveniently drawn to better explain the efficiency concept, relationship between the input and output powers were shown using an equivalent circuit that includes power losses.

Step five deduced the condition to operate in CCM. An inequality equation was deduced to easily establish the limit in which the converter operates in CCM.

Simulation results consolidated the acquired concepts during the methodology showing the variations of voltage gain and efficiency when losses are changed. Also, CCM limit was clearly shown in a figure to reinforce such concept, so it was possible to define the operating range of the converter.

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1. Docente e investigador del Grupo GIMEL. Departamento de Ingeniería Eléctrica. Universidad de Antioquia. Ingeniero electricista y doctor en Ingeniería Electrónica. Email: nicolas.munoz@udea.edu.co

2. Docente e investigador del Grupo GIMEL. Departamento de Ingeniería Eléctrica. Universidad de Antioquia. Ingeniero electricista y doctor en Ingeniería Eléctrica. Email: jmaria.lopez@udea.edu.co

3. Docente e investigador del Grupo GIMEL. Departamento de Ingeniería Eléctrica. Universidad de Antioquia. Ingeniero electricista y doctor en Ingeniería Eléctrica. Email: fernando.villada@udea.edu.co

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