

# A comparison study of MPC strategies based on minimum variance control index performance

## Comparación de estrategias MPC basado en índice de mínima varianza

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#### ABSTRACT:

Model Predictive Control (MPC) is a useful tool when controlling processes that handle a large number of input and output variables. This study presents a comparison of different MPC strategies when they are subjected to control process variables directly. The strategies studied are IMC, GPC, MPC-D, MPC-DR, and DMC. Evaluation of the performance of the controlled loop was performed with the filtering and correlation analysis algorithm (FCOR). The methodology proposed is validated in a Continuous Stirred-Tank Reactor (CSTR) case study. Discrete predictive control demonstrated the best results in this study.

**Keywords:** MPC design, Minimum variance control, FCOR, CSTR

#### RESUMEN:

El Control predictivo de modelos (MPC) es una herramienta útil para controlar procesos que manejan un gran número de variables de entrada y salida. Este estudio presenta una comparación de diferentes estrategias de MPC cuando son usadas para controlar directamente variables de proceso. Las estrategias estudiadas son IMC, GPC, MPC-D, MPC-DR y DMC. La evaluación del desempeño del lazo de control se realizó con el algoritmo de análisis de filtrado y correlación (FCOR). La metodología propuesta se valida en un caso de estudio tipo CSTR. El control predictivo discreto demostró los mejores resultados en este estudio.

**Palabras clave:** Diseño MPC, Control de Mínima Varianza, FCOR, CSTR

## 1. Introduction

Along history there have been countless industrial incidents that have resulted in several loss of human lives, as well as billions of economic losses associated with these events. Bhopal disaster is the world's worst industrial disaster and took place in a pesticide plant in Bhopal. That disaster occurred in 1984, and at least 3.000 people died in the first 24 hours

and later 15.000 more because of the aftermath of the event (Zio & Aven, 2013). This and many other incidents (Brice, 2008; Zio & Aven, 2013) have been caused for an unidentified (or not) failure in the process, which caused a sequence of 'inter-dependent' errors and ended in an unfortunate event.

These failures might have been prevented if one could keep constant monitoring of the control variables associated with the critical process in industrial plants. As a matter of fact, on average, a control engineer is responsible for about 450 control loops; however, the number of loops ranges between 30 and 2.000 (Bauer, Horch, Xie, Jelali, & Thornhill, 2016). Moreover, at the same time, this control engineer has to take care of administrative tasks and non-related to process control (Bauer et al., 2016).

A control engineer's primary purpose when designing and implementing a control loop must be to guarantee and stable operation and an efficient production (Lindström, Kyösti, & Delsing, 2018; Sanjuan, Kandel, & Smith, 2006). With the intention of contributing to the control area, this work has been divided in two parts, first, the design and implementation of Model Predictive Controllers (MPC) are considered and second, two index performances are proposed in order to compare the MPC controllers mentioned previously. These indexes are the Filtering and Correlation method (FCOR) and the Integral of Absolute Error IAE (IAE), for a continuous production process, which is a reactor with heating through a coil with steam, where two reactions take action. A more detailed description of the process can be found in (Sanjuan et al., 2006).

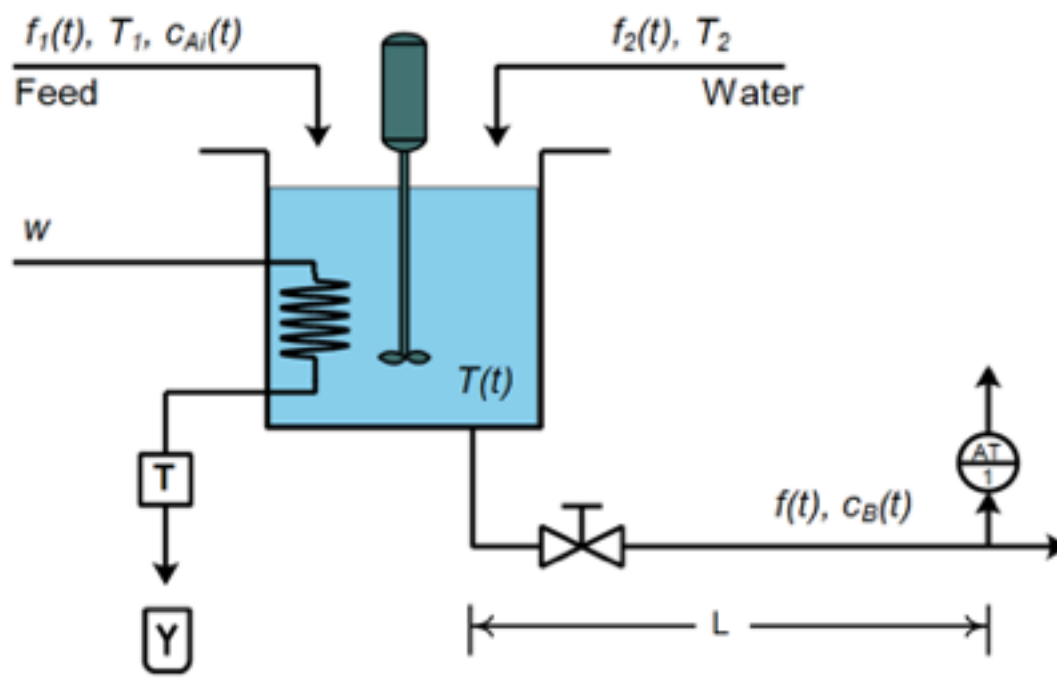
The strategies implemented in this work are based on IMC (Duarte et al., 2017; Garcia & Morari, 1982), GPC (Clarke, Mohtadi, & Tuffs, 1987) and MPC Discrete (Wang, 2009) methods. In the implementation of these strategies the following considerations were taken:

- Output variables: outlet stream concentration, product temperature, and the outlet flow stream,  $y$ , respectively.
- Inlet variables: feed flow, water flow, and steam flow,  $u$ , and  $v$ , respectively.
- Systems implemented must be MIMO 2x2.
- There are often reports about sudden variations in the reactor's inlet water pressure.

Results have shown that the best performance was achieved using control systems based on discrete time (MPC-D and MPC-DR). The worst performance was presented by control systems based on GPC and DMC. In the present work two index performances are proposed as metrics of the behavior or accomplishment of the objectives of a control loop in an industrial process, taken as a case study a CSTR which is a typical process that can be found in industrial practice (dos SANTOS & others, 2016; Rivera, Alzate, & Arias, 2015).

## 1.1. Process Model

**Figure 1**  
Continuous Stirred-Tank Reactor (CSTR),  
taken from (Sanjuan et al., 2006)



The process to be controlled is presented in Figure 1. The process involves a reactor with heating through a coil with steam, where the following two reactions take place simultaneously:



Reaction (1) is endothermic with a heat of reaction obtained from the coil, while the heat for the reaction (2) is negligible.

Reaction rates are given by

$$\begin{aligned} r_A(t) &= k_0 C_A(t) C_B(t) e^{-\frac{E}{RT(t)}} \\ r_C(t) &= k'_0 C_A(t) C_C(t) \end{aligned} \quad (1.2)$$

On the other hand, the outlet valve flow is given by

$$f(t) = C_v \sqrt{h(t)} \quad (1.3)$$

The densities in the feed stream, the reactor, and the output stream are a function of the concentration of B, this is

$$\rho(t) = \rho_0 + dC_B(t) \quad (1.4)$$

The manager of the plant has indicated that one of the primary sources of disturbance is the concentration in the feed stream,  $C_{Ai}(t)$ .

Below are established the different balances for the system:

**Global Mass Balance:**

$$A \frac{d}{dt} (h(t)\rho(t)) = \rho_1(t)F_1(t) + \rho_2(t)F_2(t) - \rho(t)F(t) \quad (1.5)$$

**Energy Balances:**

*Reactor*

$$AC_v \frac{d}{dt} (h(t)\rho(t)T(t)) = c_{p1}\rho_1 T_1 F_1(t) + c_{p2}\rho_2 T_2 F_2(t) - c_p \rho(t) T(t) F(t) + U_o A_c (T_c(t) - T(t)) - \Delta H_{rA} A r_A(t) h(t) \quad (1.6)$$

*Coil*

$$C_{pc} w_c L_c \frac{d}{dt} (T(t)) = \lambda W(t) - U_o A_c (T_c(t) - T(t)) \quad (1.7)$$

**Mass balances by component:**

*A component*

$$A \frac{d}{dt} (h(t)C_A(t)) = C_{Ai}(t)F_1(t) - C_A(t)F(t) - A r_A(t) h(t) \quad (1.8)$$

*B component*

$$A \frac{d}{dt} (h(t)C_B(t)) = -C_B(t)F(t) + 0.5 A r_A(t) h(t) + 0.5 A r_C(t) h(t) \quad (1.9)$$

*C component*

$$A \frac{d}{dt} (h(t)C_C(t)) = -C_C(t)F(t) + 0.5 A r_A(t) h(t) - A r_C(t) h(t) \quad (1.10)$$

Equations (1.1) through (1.10) constitute the dynamic model for the process study.

## 1.2. Instrumentation

The characteristics of the different transmitters necessary to carry out the process control were determined from the specification sheets of instruments available in the market, such as:

The level and flow transmitters have quite low response times therefore, they were only modeled as pure gains.

For the temperature sensor, a range of 200°F was selected, with a time constant of 0.5s.

A maximum concentration for B of 2 was defined, then taking as span 2. For this type of samples, the retention times in the columns are approximately 4 min, which leads to a time constant of 0.8 min.

Typically the steam is worked as compressible fluid, although for the conditions established in the problem where the pressure and the saturation temperature are close to atmospheric (17.02 psi and 219.45 F) a small pressure drop across the valve is considered and for these conditions the sizing of the valve can be approximated to a liquid service fluid valve. Twice as much of the flow in steady state was taken for the maximum steam flow and a maximum pressure drop of 4 psi and a minimum of 2 psi (which can approximate the behavior of a valve for fluids service), then was calculated. The a value was obtained to determine which type of valve had to be chosen, the value obtained was quite low (a value of 2.4); therefore, it was chosen proportionally, a gain of 0.589 was obtained. Similarly, the valves of the feed and water streams were dimensioned.

The characteristics of the transmitters and final control elements used in the simulations are summarized below.

**Table 1**  
Characteristics of the Transmitters and Final Control elements

<b>Process Stream</b>	<b>Instrument</b>	<b>Gain</b>	<b>Time Constant</b>
Feed Stream	Flow Transmitter	2.0544	
Water Stream	Flow Transmitter	2.7650	
Outlet Stream	Concentration Transmitter	50	0.8
CSTR Level	Level Transmitter	4	
Outlet Stream	Temperature Transmitter	0.5	0.0017
Steam Stream	Steam Valve	0.5893	0.1667
Water Stream	Water Valve	2.7056	0.1667
Feed Stream	Feed Valve	3.6414	0.1667

### 1.3. Process Analysis

### 1.3.1. Selection of Variable-Couples Manipulated / Controlled Variable

Since the design specifications state that MIMO 2X2 control strategies must be implemented, it was decided to control the concentration and temperature; and to determine the most appropriate manipulated variables for each controlled variable, the steady-state relative gain arrangement (RGA) was calculated (Bosgra, 2007). The RGA obtained was:

$$\begin{bmatrix} h \\ T \\ C_B \end{bmatrix} = \begin{bmatrix} 0.5173 & 0.4837 & -0.0011 \\ 0.0228 & -0.0647 & 1.0419 \\ 0.4598 & 0.5810 & -0.0408 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ w \end{bmatrix} \quad (1.11)$$

Then, based on (1.11) it was decided to manipulate  $f_2$  to control the concentration  $C_B$ , and  $w$  to control the temperature  $T$ . For the case of the level, it is clear that the best option would be to manipulate the feed flow,  $f_1$ .

### 1.3.2. Linear Model

Using the procedure Fit 3 described in (Smith & Corripio, 1985), the following matrix of transfer functions was obtained:

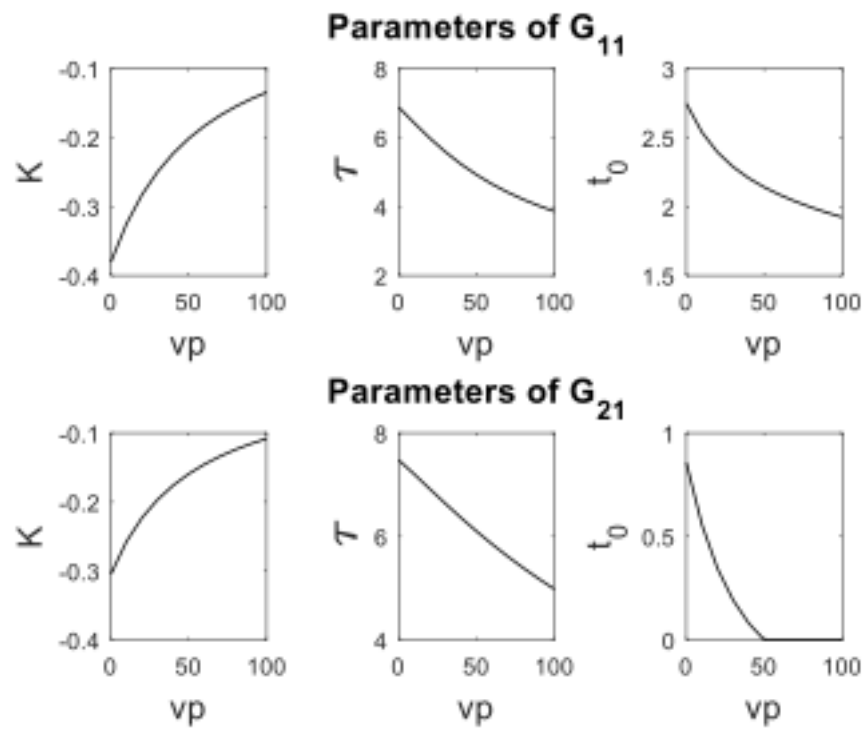
$$G(s) = \begin{bmatrix} \frac{-0.2828 e^{-2.39s}}{5.984s + 1} & \frac{-0.01798 e^{-2.95s}}{5.939s + 1} \\ \frac{-0.229 e^{-0.345s}}{6.918s + 1} & \frac{0.1398 e^{-0.394s}}{6.151s + 1} \end{bmatrix} \quad (1.12)$$

Where  $G(s) = \frac{C(s)}{M(s)}$ ,  $C(s) = [C_s(s) \ T(s)]^T$  is the vector of controlled variables and  $M(s) = [VP(s) \ VP_w(s)]^T$  is the vector of manipulated variables (water valve position and steam valve position). The vector of manipulated variables in steady state is  $\bar{m} = [38.7298 \ 70.7107]$ . In (1.12) it can be observed that at the selected operating point, the water flow effects in a similar manner both controlled variables, while the effect of the steam flow is reflected mostly in the temperature. Therefore, it is to be expected that any variation in  $f_2$  will generate undesirable effects on the temperature.

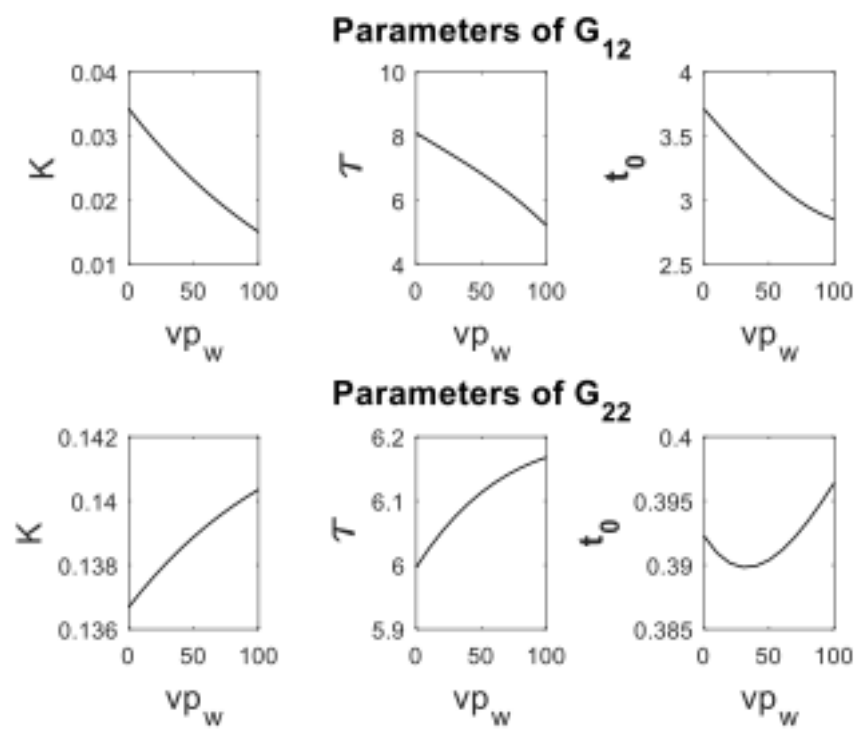
### 1.3.3. Nonlinearity Analysis

Figure 2 (Figure 3) shows the behavior of the parameters of the transfer functions  $G_{11}(s)$  and  $G_{21}(s)$  ( $G_{12}(s)$  and  $G_{22}(s)$ ) as the position of the water valve (steam valve) varies from 0 to 100%. From Figure 2 it can be inferred that as the percentage of opening of the valve increases, the controller (tuned in  $vp = 38.7298$ ) will behave aggressively, otherwise, slow response is expected. Now, given the negative value of the gain in  $G_{11}(s)$ , a reduction in  $vp$  would be expected when it is necessary to increase  $C_B$  and vice-versa. An aggressive response has the advantage that response times will be reduced; however, valve saturation and instabilities might be achieved. On the other hand, in the case of a slow response, although instability would not be expected to be a problem, the response times would be longer and could have greater overshoots. As regards the interactions,  $G_{21}(s)$ , when increasing  $vp$ , the effect of them is reduced (lower absolute value of  $k$ ) and vice-versa.

**Figure 2**  
Process parameters of  $G_{11}(s)$  and  $G_{21}(s)$  vs  $vp$



**Figure 3**  
Process parameters of  $G_{12}(s)$  and  $G_{22}(s)$  vs  $vp_w$



In this case, no considerable variations were observed in the parameters of  $G_{22}(s)$ , which was to be expected given the linear relationship between the steam flow and the temperature (1.7); therefore, similar performances are expected throughout the range of variation of  $vp_w$ .

## 2. Control Strategies

### 2.1. Internal Model Control

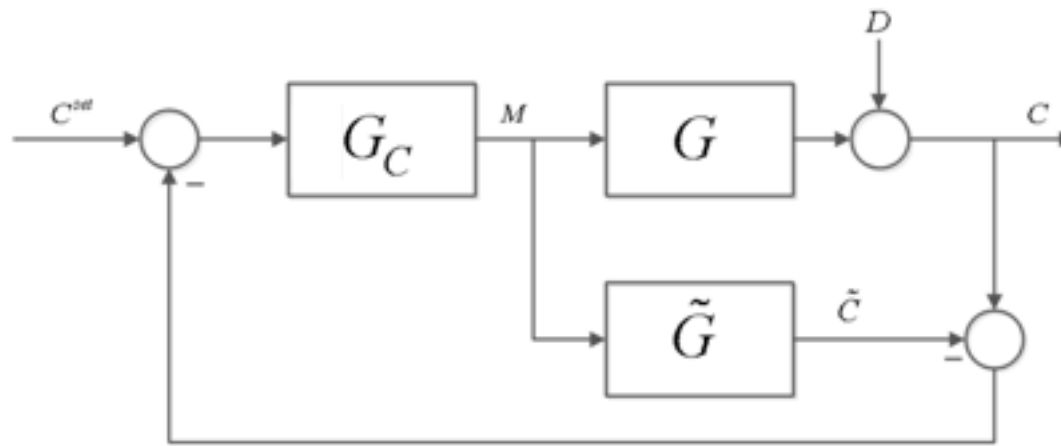
Figure 4 shows the block diagram of the IMC strategy implemented, (Garcia & Morari, 1982).  $G$  is the process to be controlled (equations (1.1) through (1.10)),  $\tilde{G}$  is the design model of that process, (1.12),  $G_C$  in the controller IMC,  $C^{SET}$  is the vector of set points,  $C$  is the vector of controlled variables,  $M$  is the manipulated variables vector and  $D$  is the disturbances vector.  $\tilde{G}$  can be decomposed in an invertible part,  $\tilde{G}_-$ , and a non-invertible part,  $\tilde{G}_+$ , as follows

$$\tilde{G} = \tilde{G}_- \tilde{G}_+ \quad (2.1)$$

Where, in this particular case,

$$\tilde{G}_-(s) = \begin{bmatrix} \frac{-0.2828}{5.984s + 1} & \frac{0.01798}{5.939s + 1} \\ \frac{-0.2229}{6.918s + 1} & \frac{0.1398}{6.151s + 1} \end{bmatrix} \quad (2.2)$$

**Figure 4**  
IMC Block Diagram Strategy



Then, the controller is given by

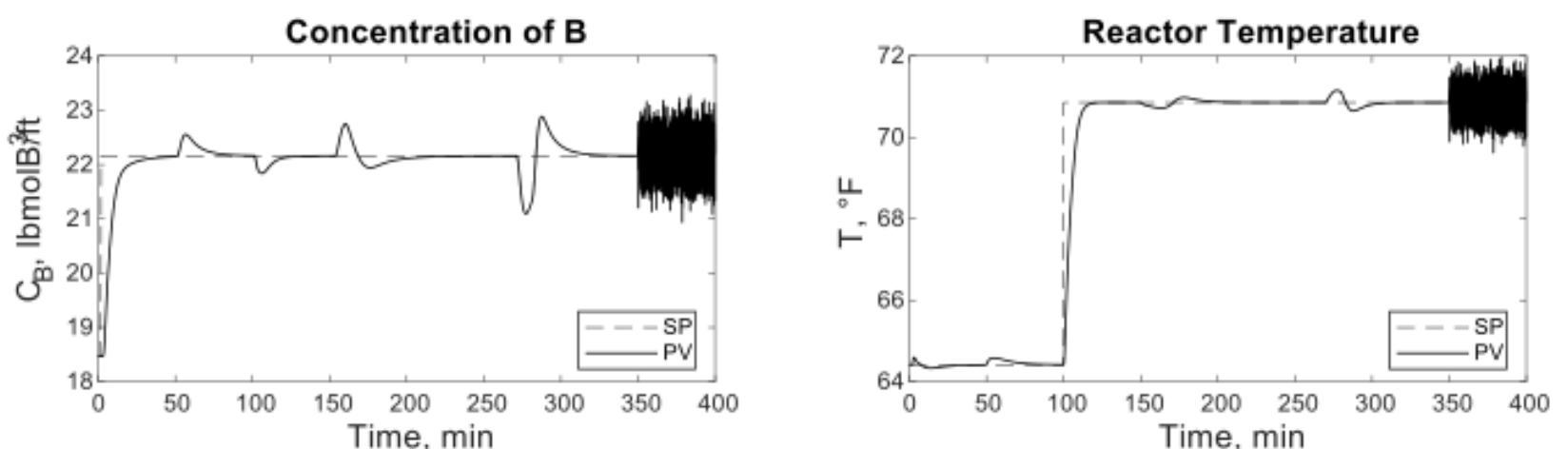
$$G_C = \tilde{G}_-^{-1} G_f \quad (2.3)$$

Where, in this particular case,  $G_f$  was chosen as:

$$G_f(s) = \text{diag} \left( \frac{1}{5s + 1} \quad \frac{1}{5s + 1} \right) \quad (2.4)$$

Figure 5 shows the results obtained are presented under the following operating conditions. A 20% increase in the setpoint of  $C_B$  and 10% increase in the setpoint of  $T$ , at  $t = 2$  min and  $t = 100$  minutes respectively. A 20% reduction in the pressure drop of the water valve at  $t = 50$  minutes, a closing of the reactor discharge valve of 30% between minutes 150 and 160. Finally, a 10% drop in the concentration of A between minutes 270 and 280 and a Gaussian noise of zero mean in the 350th minute.

**Figure 5**  
IMC Closed-Loop System Response



## 2.2. Generalized Predictive Control Based on Transfer Functions, GPC-FT

The GPC control is based on the use of a CARIMA model (Controlled Auto-Regressive Integrating Moving Average) to predict the behavior of the process in a determined prediction horizon. This model is presented in the equation (2.5).

$$A(z^{-1})y_k = z^{-d}B(z^{-1})m_{k-1} \quad (2.5)$$

To determine the values predicted by this model, the recursive calculation can be performed from the past values of the controlled variable. Alternatively, also solve the Diophantine equation shown in equation (2.6)

$$1 = E_j(z^{-1})\tilde{A}(z^{-1}) + z^{-j}F_j(z^{-1}), \quad \tilde{A}(z^{-1}) = \Delta A(z^{-1}) \quad (2.6)$$

In this manner, the solution of this equation involves calculating the matrices  $E_j$  and  $F_j$ . An example of this solution is proposed in Camacho et al. (Camacho & Bordons, 2007). The matrix  $G_a$  and  $G_b$  can be obtained from the matrix  $E_j$  and  $B(z^{-1})$ ; therefore the prediction model is presented in equation (2.7).

$$\begin{bmatrix} \hat{y}_{k+d+1|k} \\ \hat{y}_{k+d+2|k} \\ \vdots \\ \hat{y}_{k+d+N_2|k} \\ \hat{y} \end{bmatrix} = G_a \begin{bmatrix} \Delta m_k \\ \Delta m_{k+1} \\ \vdots \\ \Delta m_{k+N_m-1} \\ m_a \end{bmatrix} + G_b \begin{bmatrix} \Delta m_{k-1} \\ \Delta m_{k-2} \\ \vdots \\ \Delta m_{k-nb} \\ m_b \end{bmatrix} + F \begin{bmatrix} \hat{y}_{k+d|k} \\ \hat{y}_{k+d-1|k} \\ \vdots \\ \hat{y}_{k+d-na|k} \\ y_b \end{bmatrix} \quad (2.7)$$

Then, using the cost function proposed by Clarke et al. (Clarke et al., 1987), it can be minimized by implementing the proposed solution in the equation (2.8).

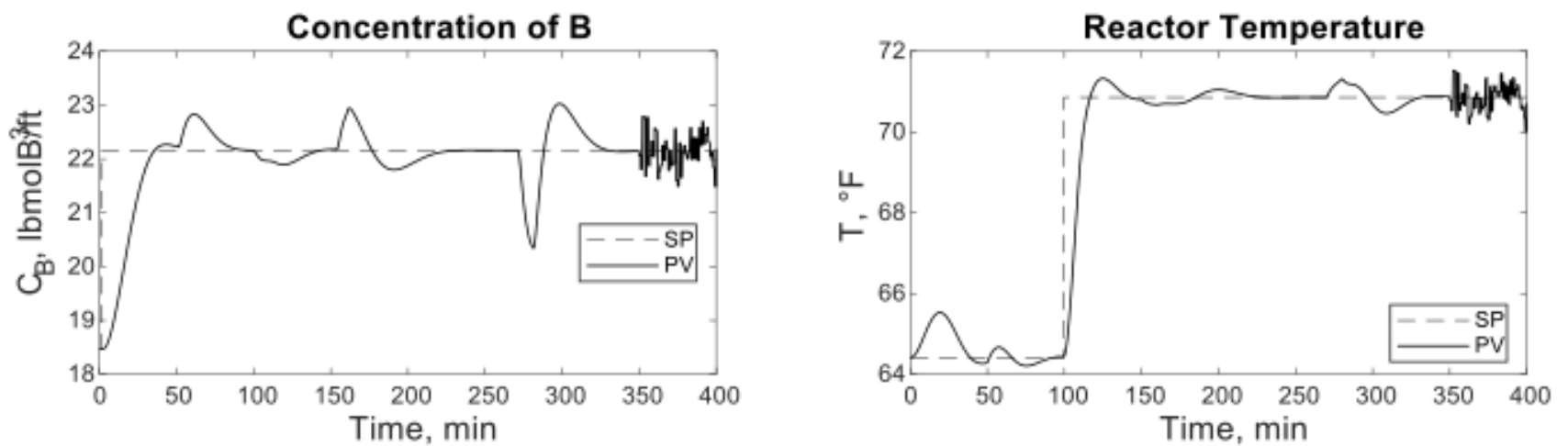
$$m = (G_a^T \delta I G_a + \lambda I)^{-1} G_a^T (\omega - fr) \quad (2.8)$$

Where  $\omega$  represents future values of the setpoint's (reference point) of the control loop and  $fr$  represents the free process response, which is stated in the equation (2.9).

$$fr = G_b m_b + F y_b \quad (2.9)$$

Responses shown in Figure 6 where obtained by applying the previously mentioned GPC control strategy on the non-isothermal reactor.

**Figure 6**  
GPC-FT Closed-Loop System Response



The tuning parameters used were prediction horizon  $N_2 = 10$ , control horizon  $N_m = 5$ ,  $N_m = 5$ ,  $\delta_1 = 1$ ,  $\delta_2 = 50$ ,  $\lambda_1 = 1.5$ ,  $\lambda_2 = 0.1$ . Figure 6 shows a little oscillatory and quite stable response, able to keep the system controlled and stable even when there is noise in the controlled variable. When comparing with the other strategies, it can be observed that it is a little slower when responding to setpoint changes.



## 2.3. Model Predictive Control Based on Discrete Time, MPC-D

Unlike the previous cases, for the implementation of this MPC algorithm an implementation was carried out on state variables in discrete time of the process (Wang, 2009). That implementation was obtained from (1.12), using PADE approximations for dead time (of the order of 3, 2, 1 and 1 for  $G_{11}(s)$ ,  $G_{22}(s)$ ,  $G_{12}(s)$  y  $G_{21}(s)$  respectively); zero order retainers and a sampling time of 1 min for the discrete time transformation. In total, the representation in state variables resulted in 11 states.

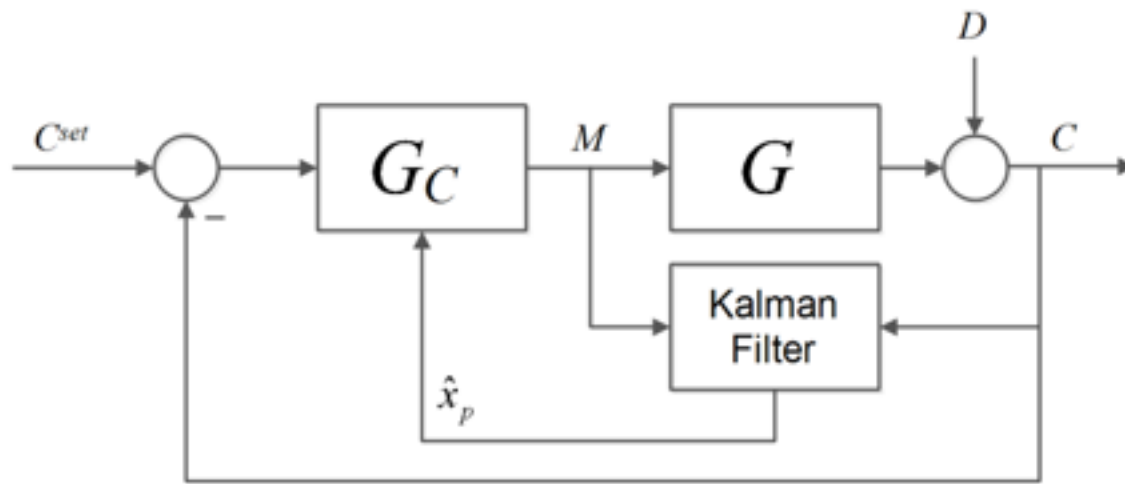
Now, with the purpose of obtaining an integral action, we work with representation in variables of extended state, of the form

$$\underbrace{\begin{bmatrix} \Delta x_p(k+1) \\ e(k+1) \end{bmatrix}}_{x_{k+1}} = \underbrace{\begin{bmatrix} A_p & 0 \\ C_p A_p & 1 \end{bmatrix}}_{A_e} \underbrace{\begin{bmatrix} \Delta x_p(k) \\ e(k) \end{bmatrix}}_{x_k} + \underbrace{\begin{bmatrix} B_p \\ C_p B_p \end{bmatrix}}_{B_e} \Delta u(k) \quad (2.10)$$

$$e(k) = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{C_e} \begin{bmatrix} \Delta x_p(k) \\ y(k) \end{bmatrix} \quad (2.11)$$

Figure 7 shows the MPC-D block diagram control system implemented, where  $G$  is the process to be controlled,  $G_C$  is the MPC-D controller.

**Figure 7**  
MPC-D Strategy Block Diagram

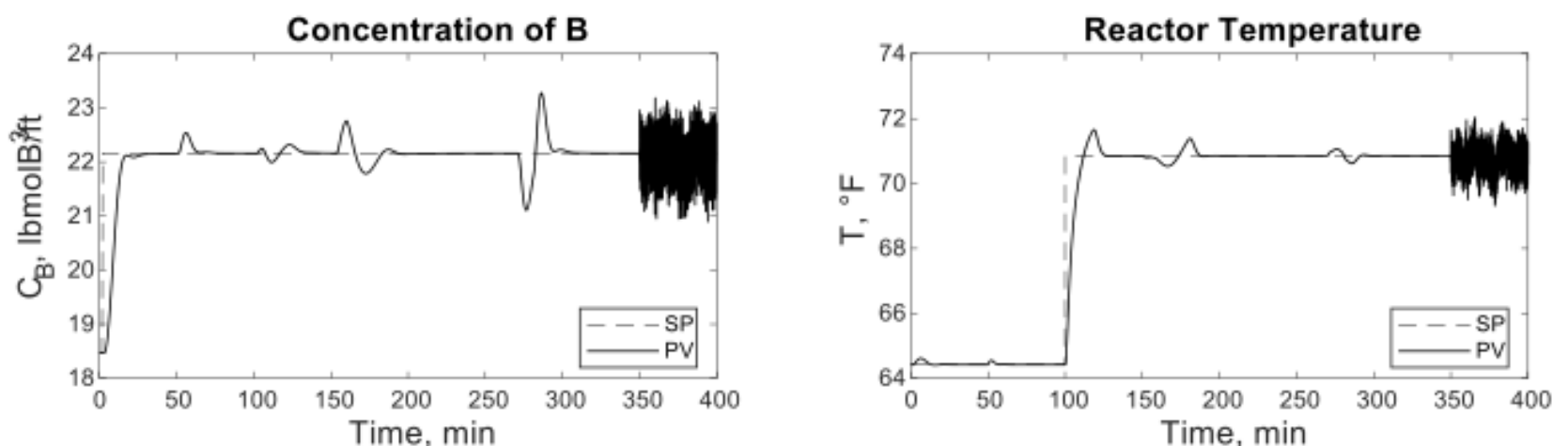


The control law implemented has the form:

$$\Delta m_k = k_{MPC-D} \hat{x}_k \quad (2.12)$$

Where  $k_{MPC-D}$  is a matrix of dimensions  $2 \times 13$ , obtained following the procedure described in (Wang, 2009), y  $\hat{x}_k$  is the vector of estimated states of the extended model (2.10 and 2.11). Figure 8 shows the response of the system in closed-loop under the same operating conditions considered for the IMC system.

**Figure 8**  
MPC-D System Response in Closed-Loop



## 2.4. Model Predictive Control Based on Discrete Time with Restrictions, MPC-DR

The control strategy implemented in this case is equivalent to that proposed by Rawlings in (Rawlings, 2000), in the sense that it starts from a representation in state variables and allows to handle restrictions in the main variables of the process. Basically, the same control system designed in the previous section is used, and  $G_R$  stage is added, responsible for recalculating the control signal in case there is a violation of the restrictions presented in Figure 9. The tuning parameters were again  $\alpha = [0.9 \ 0.8]$   $N = [4 \ 4]$ ,  $N_p = 100$ ,  $Q = C_e^T C_e$  and  $R_L = \text{diag}(0.001 \ 0.02)$ .

**Figure 9**  
MPC-DR Strategy Block Diagram

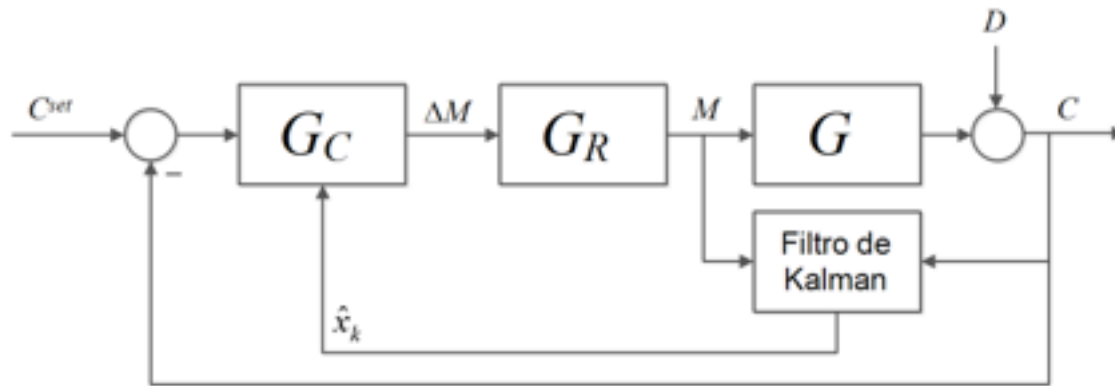
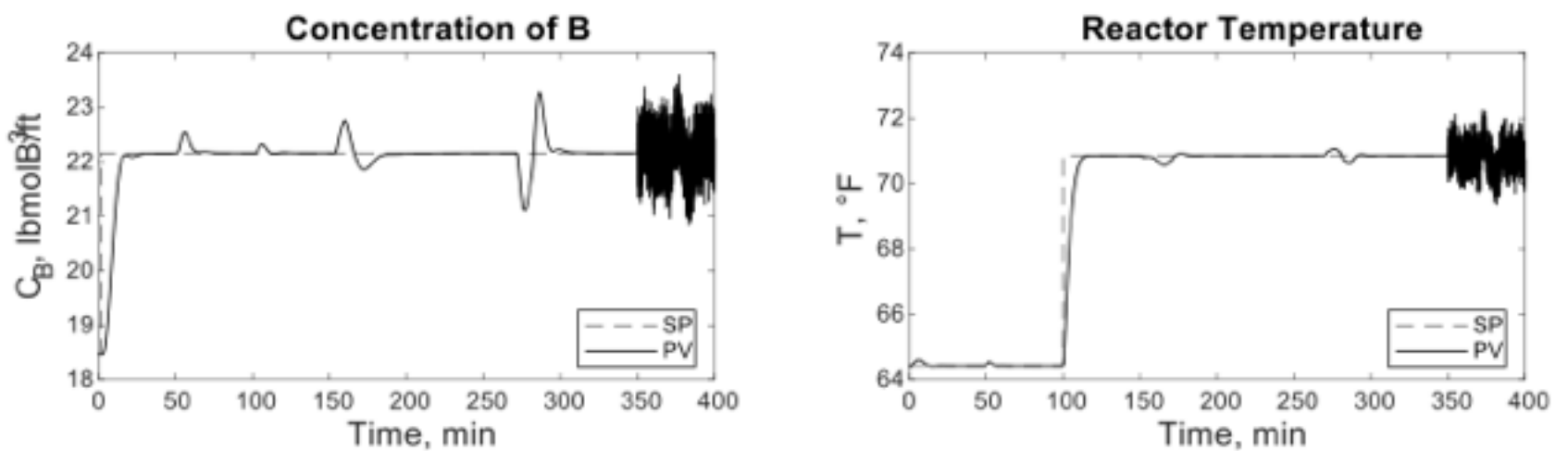


Figure 10 shows the response of the closed-loop system under the same operating conditions considered in the previous cases. It is worth mentioning that in this case restrictions were considered only in the control signal in order to maintain similar conditions with the other systems involved, in which a saturation block was used to prevent the control signals from leaving the range  $0 \leq \Delta m \leq 100$ .

**Figure 10**  
MPC-DR Closed Loop System Response with restrictions in  $\Delta m_k$



## 2.5. Dynamic Matrix Control

This strategy of predictive control, unlike the other strategies, does not use an explicit model within its structure to determine the future behavior of the controlled variable (s). In this sense, this approach is advantageous, since to obtain the prediction it is only necessary to have the response curve of the process and assume a linear behavior. In comparison with the other strategies, assuming linearity is not a disadvantage because all the strategies used to make this assumption. Then, from the response vector, the unit response vector is obtained. This vector is calculated by subtracting, at each of the values of the response, the initial value of this. To then divide the result by the value of the used to obtain the response vector. Thus, the response matrix is represented in **(2.13)**.

$$A = \begin{bmatrix} A_{v1} & 0 & 0 \\ A_{v2} & A_{v1} & 0 \\ \vdots & A_{v2} & A_{v1} \\ \vdots & \vdots & \vdots \\ A_{vPH} & A_{vPH} & A_{vPH} \end{bmatrix} \quad (2.13)$$

For a  $2 \times 2$  control strategy, the  $A$  matrix must be constructed according to the representation shown in (2.14).

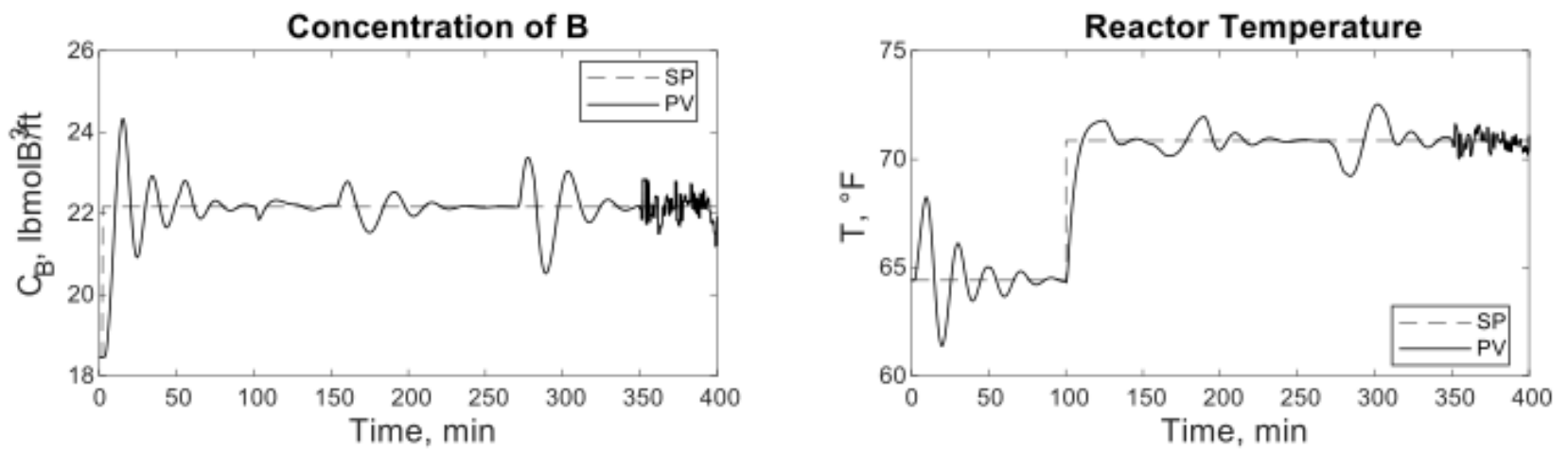
$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad (2.14)$$

Then, applying the least squares solution, and using tuning parameters, we have that the control law is represented in equation (2.15).

$$\Delta m = \left( [A^T : L_l] \begin{bmatrix} A \\ L_l \end{bmatrix} \right)^{-1} [A^T : L_l] E \quad (2.15)$$

Finally, Figure 11 shows the performance of the control strategy in the process under study.

**Figure 11**  
DMC Closed Loop System Response  $m_k$

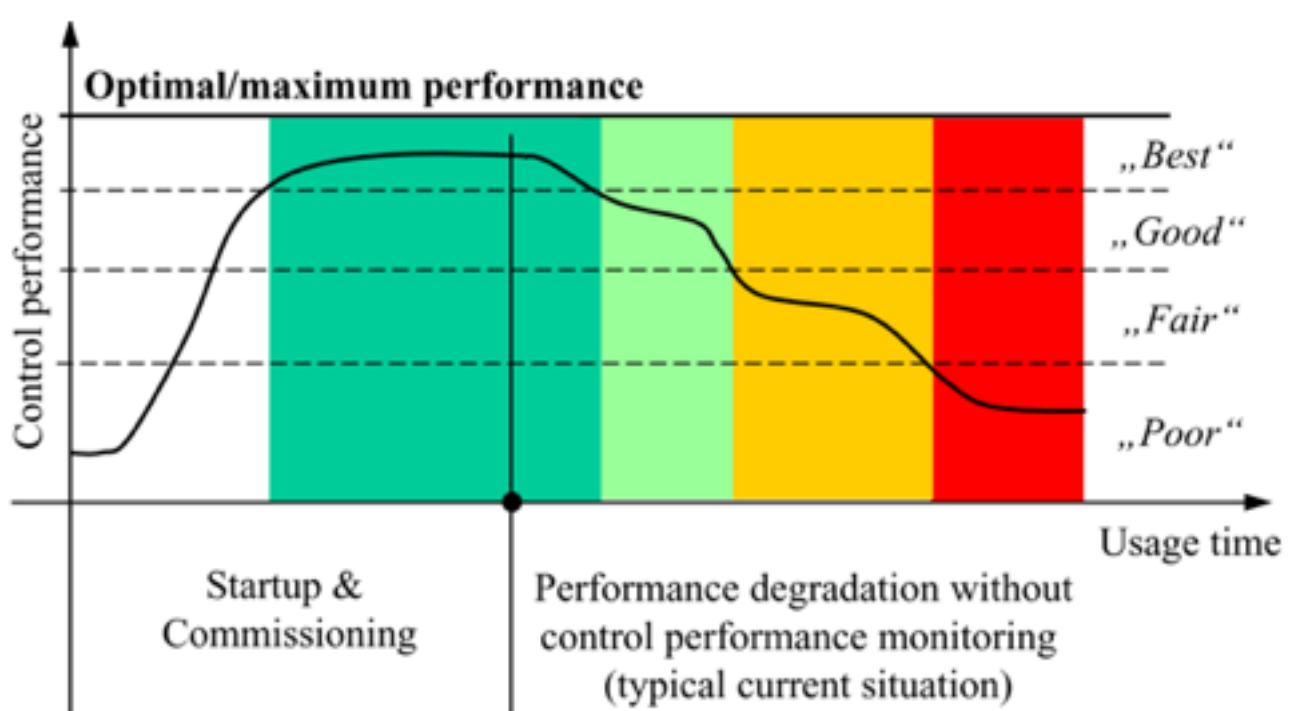


The tuning parameters used were  $\lambda_1 = 1$  and  $\lambda_2 = 1.5$ . It can be seen a quick response and a bit more aggressive than those obtained in the previous strategies; however, the control loop is stable at all times, even when there is noise in the control signal.

### 3. Control Performance Index

In an industrial process, there are typically hundreds to thousands of controllers, which are usually of the Proportional-Integral-Derivative (PID) type, but there may also be non-linear, adaptive, or multivariable-predictive controllers (Harris, Seppala, & Desborough, 1999). However, most controllers work well during the first stage of operation (typically the first six months) (Jelali, 2012), after which their performance begins to deteriorate gradually until they are finally destined to be manual. See Figure 12.

**Figure 12**  
Typical decay of industrial process control performance due to different factors, taken from (Jelali, 2012)



The methods for evaluating control loop performance are divided into: 1. Deterministic (based on settling time, based on area, performance indices), 2. Advanced (based on a model: Gaussian linear quadratic LQG, minimum generalized variance GMV, predictive models), and 3. Stochastic (based on data); the latter can be classified as 3.1 specified by the user (desired behavior of the closed loop, reference model, historical), 3.2 First-Pass (descriptive statistics, auto-correlation, spectral analysis) 3.3 Minimum variance (MV) (based on the interacting matrix, not based on the interacting matrix). Several interests in the performance of the plant or the process has been shown in recent studies (CANO, BOTERO, & RIVERA, 2017; JORDÃO, Neto, & others, 2016; Mauricio Johnny & RODRIGUEZ, 2015) , where from a different perspective, they look forward the improvement of the overall system. The methods for evaluating the performance of the control loop are based on determining the variance of the process and comparing it with some "ideal" value or desired value. In the present study, the Filtering and Correlation algorithm (FCOR) (Huang, 1998) have been used, and it defines the key performance parameter as:

$$\eta = \frac{\sigma_{mv}^2}{\sigma_y^2} \quad (3.1)$$

Where

$\eta$ : Is the performance index, that is, how much the process can be improved with control?

$\sigma_y^2$ : It is the variance of the data.

$\sigma_{mv}^2$ : It is the minimum variance of the process, which can be calculated as follows:

$$\sigma_{mv}^2 = \left( \sum_{l=0}^{d-1} f_l^2 \right) \sigma_e^2 \quad (3.2)$$

Where  $\sigma_e^2$  is the noise variance (residuals or error if the model is being adjusted),  $d$  is the time delay (time delay) of the process and  $f_l$  are the coefficients of the infinite response model for the closed-loop process.

The infinite response model is a consequence of Wold's decomposition theorem and can be written as:

$$y_t = \left( \sum_{l=0}^{\infty} f_l z^{-l} \right) e_t \quad (3.3)$$

The previous model can be obtained through a standard transfer function of the process (3.1) and by using the long division or the command "impz" in MATLAB®, for example:

$$y_t = \frac{G_l}{1 + G_p G_c} e_t \quad (3.4)$$

The above equation can be estimated from routine operation data of the process, through a time series analysis (time-series analysis). The analysis of the performance of the process was made based on the following procedure (Huang & Kadali, 2008):

- Record the information of the variables of interest of the system in the closed-loop appropriately, without changes of setpoint. The mean of the variable must be subtracted.
- Obtain a system model (by analyzing a series of data)
- Estimate the dominant time delay of the process.
- Calculate the coefficients of the infinite response of the model in closed loop.
- Estimate the variance of the residuals.
- Calculate the minimum variance using equation (3.2).
- Estimate the current variance.
- Calculate the performance index using equation (3.1).

## 4. Results

As mentioned before, the operational conditions in consideration in the simulations are: 20% increment in setpoint and also an increment of 10% in  $\rho$ , both at  $t=2$  and  $t=100$  minutes, respectively. A pressure drop reduction of 20 % in the water valve at  $t=50$  minutes, a 30% closing in the reactor's discharge valve between minutes 150 and 160; a 10% drop in concentration of reactive A between  $t=270$  minutes and  $t=280$  and a Gaussian noise of zero mean at  $t=350$  minutes.

According to results shown in Table 2, one can observe that control strategy MPC-D is the one that permits obtain the best performance of the process concerning minimum variance when it is subjected to several disturbances as setpoint changes, variations in inlet reactive concentration, and variations in the opening of the discharge valve. The previous situation is most likely because of the Kalman-filter structure used in the control law, where there is a predictor module, and a corrector module, therefore this control strategy output will be the required for counteracting changes in the least aggressive and faster possible fashion. On the other hand, it can be evidenced that control signal changes can get to be very

sudden/abrupt, situations that could get to be counterproductive regarding the correct functioning of control valves.

**Table 2**

Summary of the performance obtained with the different strategies

	$\eta$	$IAE_{CB}$	$IAE_T$
IMC	0.1046	80.03	55.68
MPC-D	0.9302	68.83	58.89
MPD-DR	0.9214	68.31	50.91
DMC	0.4282	107.1	169.8
GPC	0.267	148.2	124.1

As for variations with respect to changes in the closure of the discharge valve it was concluded that control strategies IMC, MPC-D and MPC-DR presents the best performance, when taking into account that control variables are at their desired value or *setpoint*, as well as for 'stability' or 'behavior' of the controller output. Regarding the behavior of the control strategies when comparing the total IAE value, it was concluded that the best performance was obtained using the MPC-DR control strategy.

On the contrary, strategies like GPC-TF or DMC presents a lower performance. Therefore, DMC strategy presents the most significant variations against setpoint changes, and the GPC-TF control strategy presents the most considerable variations against sudden changes in the inlet water pressure. Regarding inlet reactive concentration changes, both strategies present a similar behavior between them.

## 5. Conclusions

Considering that the effects due to changes in setpoints over other output variables (for example, the effect on due to a change in ) are indicative of the level of interaction between inputs and outputs, it can be concluded that the internal model control (IMC) is the one that best dissociates interactions achieved; however it presents a very low-performance index.

Control strategies based on discrete MPC presents the best behavior against inlet disturbances, this results evident because their response was the best against changes in the water supply flow pressure, as well as a performance index close to 1.

The worst performance regarding the system and operational conditions analyzed was given for the GPC-TF and MPC-DR control strategies.

The best performance under the operational conditions mentioned above and also taking into account both performance indexes was presented by the MPC-DR scheme, mainly because of a faster speed response. It is important to mention that the performance index helps us determine how good or how bad the control loop have performance under some disturbances in a simpler form (compare data in Table 2) vs. traditional manner (compare Figures 5,6,8,10, and 11). These results very usefully when monitoring control loops in an industrial environment, where it can be found between one hundred or more than a thousand control loops. However, as in this research is evidenced, it is very beneficial having more than one performance index (for example, IAE besides of ) so one can be able to have a better understanding of the behavior of the control loops, and then from this information being able to take better decisions for different tasks: re-tuning or re-configuration of the control strategies.

It should be noted that, although the behavior accomplishes with IMC is similar to MPC-DR, and the additional advantage of the last one is that since it is a strategy based on discrete time, its implementation in a device like a PLC is more straightforward. It is important to mention that IMC strategy does not allow an effective integration with the use of



restrictions, both in manipulated and controlled variables and then depending on the process to control this may not be the most viable option, although it can draw attention to the industrial level due to its ease of implementation.

As for the other predictive control strategies studied, it is important mentioning that an imperative step for having good performance is when setting the tuning of the controller, so according to the present study, it might be possible to be necessary to carry out an optimization process that allows obtaining the best possible tuning in each case.

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